A Generic Model of Equipment Availability under Imperfect Maintenance

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Abstract

We explore the impact of imperfect repair (as described by Kijima's first virtual age model) on the availability of repairable equipment. Simulation modeling is used to evaluate equipment availability, and a generic availability function is proposed. Numerical results indicate that our proposed function provides a reasonable approximation of equipment availability, which simplifies meaningful analysis for the unit. Therefore, a method is defined for determining optimum equipment replacement intervals based on average cost. In our presentation, we will also describe our efforts to build meta-models to convert equipment reliability and maintainability parameters into the coefficients of the availability model and eliminate the need to perform simulation to obtain the parameters of the availability model.

1. Introduction

The majority of equipment used in manufacturing and other industrial applications can be and is often repaired upon failure. The use of mathematical modeling for evaluating, improving and optimizing the performance of repairable equipment is well established in the literature. A key feature of these models is the assumption regarding the influence of repair on equipment aging. The vast majority of these models assume either perfect repair (renewal) or minimal repair. Perfect repair implies that the equipment is "good as new" after repair. Minimal repair implies that the equipment is "bad as old" after repair, i.e. the equipment has the same age as it did at the time of failure. Recently, more attention has been given to the concept of imperfect maintenance (Pham and Wang, 1996). Imperfect maintenance includes a wide variety of models that assume repair is somewhere between perfect repair and minimal repair. Virtual age models (Kijima 1989, Kijima *et al.* 1988) are one subset of imperfect maintenance models. The term virtual age is used to avoid confusion between the age of the equipment and the elapsed operating time.

Kijima's first virtual age model (Kijima 1989), the imperfect maintenance model used in this research, can be summarized as follows. Consider a unit of equipment that, at any point in time, is in one of two states, functioning or failed (under repair), and assume that the unit is initially (at time $t = 0$) functioning. Let X_n denote the length of the n^{th} period of equipment function, and let V_n denote the virtual age of the unit at the time of the n^{th} repair completion. The virtual age model is

$$V_n = V_{n-1} + aX_n \tag{1}$$

where a is some constant such that $0 \le a \le 1$ and $V_0 = 0$. The unit accumulates age during each period of operation $\{X_1, X_2, \}$. After each failure, repair "removes" some of this accumulated age. Thus, $1-a$ captures the degree of equipment restoration achieved through repair.

Consider a unit of equipment that is required to operate on a continuous basis for a useful life of L hours. Suppose that at any point in time, the unit is in one of two states, functioning or failed (under repair), and that the unit is initially functioning. Let X_n denote the length of the n^{th} period of equipment function, and let V_n denote the virtual age of the unit at the time of the n^{th} repair completion. We use a slight modification of the Type I Kijima repair model shown in equation (1).

We assume that X_1 is a Weibull random variable having shape parameter β and scale parameter η. Thus,

$$F_1(x) = 1 - \exp\left[-\left(\frac{x}{\eta}\right)^{\beta}\right] \tag{2}$$

Since the majority of repairable equipment demonstrates increasing failure rate (IFR) behavior, we consider values of β that are greater than 1. In general, we assume that X_n has a residual Weibull distribution (with the same parameters) given survival to age V_{n-1}. We assume that the time required to complete repair is a constant value of t_r and that preventive maintenance is not performed on the unit.

2. Equipment Performance Modeling

The initial objectives of this study are: (1) to construct a discrete-event simulation model which can be used to estimate equipment availability, (2) to use the results of the simulation analysis to propose a generic availability function for such units, (3) to apply regression to the output of this simulation model to estimate the parameters of the equipment availability function, $A(t)$, and (4) to use this model to develop optimal equipment replacement strategies. The simulation model, constructed in Visual Basic, mimics the function, failure and repair of the equipment and collects availability data on the unit during its useful life.

Figure 1 contains an example plot of the estimates of the availability function at the data collection points. Since the relationship appears to be approximately exponential, a plot of $\ln(-\ln A(t))$ vs. $\ln(t)$ is created (see Figure 2).

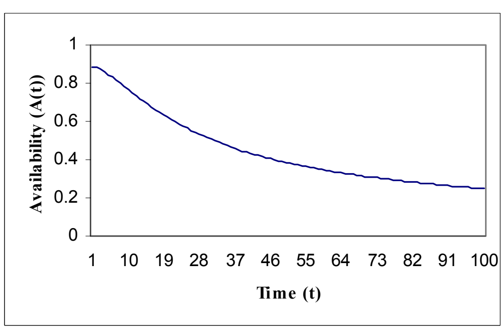

Figure 1. Sample Availability Plot

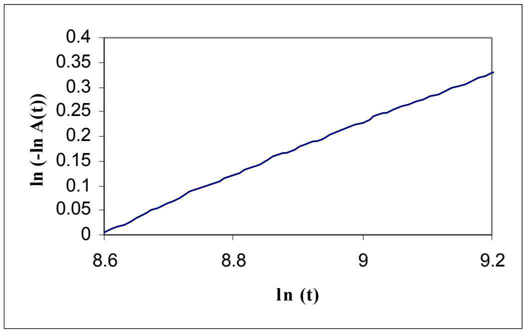

Figure 2. ln(−ln $A(t)$) vs. ln(t)

In an effort to capture the availability behavior seen in the simulation output, we define a generic availability function for units of this type. This function must be defined such that $A(0)=1$, $A(\infty)=0$, and the exponential behavior seen in the sample plots is captured. Thus, we hypothesize that the availability is given by

$$A(t)=\exp\left(-b_0 t^{b_1}\right) \tag{3}$$

To assess the accuracy of this equation, a 2^3 factorial design is formulated with respect to three constants; β, a, and t_r. The details of this design are summarized in Table 1. All eight experiments are simulated using $\eta = 1$ and $L = 100$.

Table 1. Experiment Parameters and Values

Parameter	β	a	t_r
Low Value (−1)	1.5	0.10	0.05
High Value (+1)	3.0	0.25	0.15

These observations on availability are used to estimate the values of b_0 and b_1 for each of the eight experiments. For the eight experiments, the results indicate that the mean absolute error between the simulation output and the availability function approximation ranges from 0.2% – 4%.

Given the generic model of equipment availability, we can more easily perform meaningful analysis for the unit. For example, we can determine a cost-optimal replacement time for a unit of equipment. Let τ denote the replacement time. The average cost per unit time of equipment operation, $AvgCost\ (\tau)$ can be determined using two cost parameters, acquisition cost (c_a) and the cost per unit time of downtime (c_d).

$$AvgCost(\tau)=\left(\frac{c_a}{\tau}\right)+c_d\left[1-A_{avg}(\tau)\right] \tag{4}$$

Note that $A_{avg}(\tau)$ is the average availability over the first τ time units of equipment operation, and

$$A_{avg}(\tau)=\frac{1}{\tau}\int_0^{\tau} A(t)\,dt \approx \frac{1}{\tau}\int_0^{\tau} \exp\left(-b_0 t^{b_1}\right)dt \tag{5}$$

Unfortunately, the integral in (13) must be evaluated numerically.

Suppose we wish to identify the value of τ that minimizes average cost for the equipment corresponding to our first experiment ($\beta = 1.5$, $a = 0.10$, $t_r = 0.05$). Suppose c_a = \$10,000 and c_d = \$500 per hour. With the use of Mathematica, the average cost of unit operation can be evaluated numerically for various values of τ. In fact, a plot of average cost as a function of τ can be constructed to find the optimal value of τ (see Figure 3). For this example, the optimal replacement time is 84 hours, and the minimum average cost is \$452.20 per hour.

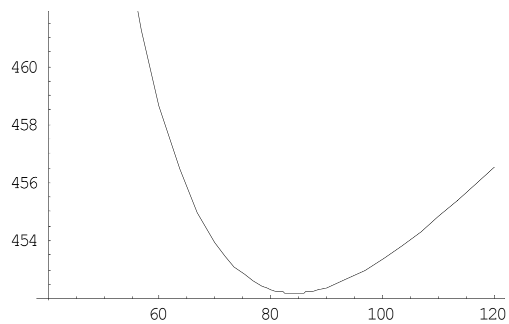

Figure 3. Average Cost vs τ

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